Instability-Enhanced Collisional Friction Can Determine the Bohm Criterion in Multiple-Ion-Species Plasmas

S. D. Baalrud,* C. C. Hegna, and J. D. Callen
Department of Engineering Physics, University of Wisconsin-Madison, 1500 Engineering Drive, Madison, Wisconsin 53706-1609, USA
(Received 10 July 2009; published 10 November 2009)

A generalized Lenard-Balescu theory that accounts for instability-enhanced collective responses is used to calculate the collisional friction between ion species in the plasma-boundary transition region (presheath). Ion-ion streaming instabilities are shown to cause such a strong frictional force that the relative flow speed between ion species cannot significantly exceed the critical threshold value (ΔV_c) at which instability onset occurs. When combined with the Bohm criterion, this condition uniquely determines the flow speed of each ion species at the plasma-sheath boundary. For cold ions, $\Delta V_c \rightarrow 0$ and each ion species leaves the plasma at a common system sound speed c_s.

DOI: 10.1103/PhysRevLett.103.205002 PACS numbers: 52.27.Cm, 52.30.Ex, 52.35.Qz, 52.40.Kh

Understanding plasma-boundary interactions requires knowing the speed at which ions leave a plasma. Determining this speed is important in a broad range of plasma applications including fusion edge plasmas [1], materials processing with plasmas [2], space plasmas [3] and diagnostics [4]. If a single ion species is present, Bohm [5] showed that ions exit the plasma with a flow speed greater than or equal to the sound speed: $V \geq \sqrt{T_e/M_i}$. It was later shown that equality typically holds [6] and this has been confirmed experimentally [7]. The Bohm criterion was later generalized to include more than one species of positive ions [8], but the result only provides a single condition for N unknowns [see Eq. (6)] where N is the number of different ion species. Thus, this criterion does not provide a unique prescription for determining the flow speed of each species as it leaves the plasma. More recent theories [9] and experiments [10–14] do not agree as to which of the possible solutions of this generalized Bohm criterion are physically realizable. In this Letter we show that ion-ion streaming instabilities, which are present in the plasma-boundary transition region, lead to an instability-enhanced collisional friction between ion species. This friction relates the relative flow speed between species and, in combination with the generalized Bohm criterion, uniquely determines the flow speed of each ion species as it leaves the plasma.

Plasmas tend to remain charge neutral and electric field free. The strong electric fields associated with boundaries are shielded to Debye-length-scale (λ_D) sheaths at the plasma-boundary interface. However, this shielding is not perfect and a "presheath" region also exists in which there is a weak electric field, but the plasma is quasineutral [5]. The presheath length is typically on the order of the ion-neutral collision length ($\lambda^{i/n}$), with a short transition region to the sheath, and the potential drop is typically $\lesssim T_e/e$ [6,7].

Theoretical works by Franklin on multiple-ion-species plasmas [9] have predicted that each species should obtain a flow speed close to its individual sound speed, $c_{s,i} \equiv \sqrt{T_e/M_i}$ at the sheath-presheath interface. However, experimental studies on two ion species plasmas in a low-temperature ($T_e \sim 1$ eV, $T_i \sim 0.02$ eV) regime have used laser-induced fluorescence [10–12] to reveal that the ion flows at the sheath edge can be far from this prediction. The measured flow speed of each species in these experiments was closer to a common "system" sound speed $c_s \equiv \sqrt{\sum_i c_{s,i}^2 n_i/n_e}$ than their individual sound speeds $c_{s,i}$.

Additional experimental evidence for a common flow speed has been provided by ion-acoustic wave measurements. Oksuz *et al.* [13] measured that for two ion species plasmas the ion-acoustic wave speed at the sheath edge is approximately twice what it is in the bulk plasma. Lee *et al.* [14] showed that this implies each ion species enters the sheath at the system sound speed c_s. However, no physical mechanism has been identified by which this solution is established, nor to motivate why the ion-acoustic wave speed doubles.

In this Letter, we show that considering collisional friction enhanced by ion-ion streaming instabilities resolves this discrepancy. For a specific example, we consider a typical plasma from the experimental literature [12]: Ar^+ and Xe^+ ions with equal densities, $T_e = 0.69$ eV, $T_i = 0.023$ eV, and a neutral pressure of 0.7 m Torr. The collisional friction contributed by Coulomb interactions alone is typically negligible in such low density and temperature plasmas; thus it has been neglected in previous theories [9]. However, instabilities can greatly enhance this friction. Noise in the MHz frequency range has been measured near the boundaries of these discharges, and this has been attributed to ion-ion streaming instabilities [11].

In the following, we calculate the instability-enhanced collisional friction force on each ion species assuming a cold ion model to describe the instability properties. For simplicity, we will consider only two ion species.

In Ref. [15] the Lenard-Balescu [16] kinetic equation $df_s/dt = \sum_{s'} C(f_s, f_{s'})$ was generalized to include convectively unstable plasmas. It was shown that the collision operator could still be written in the Landau form

$$C \equiv -\nabla_{\mathbf{v}} \cdot \int d^3v' \mathcal{Q} \cdot \left(\frac{\nabla_{\mathbf{v}'}}{m_{s'}} - \frac{\nabla_{\mathbf{v}}}{m_s}\right) f_s(\mathbf{v}) f_{s'}(\mathbf{v}') \quad (1)$$

in which $\nabla_{\mathbf{v}}$ is a velocity-space gradient and $\mathcal{Q} = \mathcal{Q}_{\rm LB} + \mathcal{Q}_{\rm IE}$ is the tensor "collisional" kernel. The Lenard-Balescu contribution describes conventional Debye-shielded Coulomb interactions:

$$\mathcal{Q}_{\rm LB} = \frac{2q_s^2 q_{s'}^2}{m_s} \int d^3k \frac{\mathbf{k}\mathbf{k}}{k^4} \frac{\delta[\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}')]}{|\hat{\varepsilon}(\mathbf{k}, \mathbf{k}\cdot\mathbf{v})|^2}. \quad (2)$$

The instability-enhanced contribution describes the enhanced collective interaction created by instabilities:

$$\mathcal{Q}_{\rm IE} = \frac{2q_s^2 q_{s'}^2}{m_s} \int d^3k \frac{\mathbf{k}\mathbf{k}}{k^4} \sum_j \frac{\gamma_j}{(\omega_{R,j} - \mathbf{k}\cdot\mathbf{v})^2 + \gamma_j^2} \times \frac{\exp(2\gamma_j t)}{[(\omega_{R,j} - \mathbf{k}\cdot\mathbf{v}')^2 + \gamma_j^2]|\partial\hat{\varepsilon}(\mathbf{k},\omega)/\partial\omega|^2_{\omega_j}}, \quad (3)$$

in which $\omega_{R,j}$ is the real part and γ_j the imaginary part of the dispersion relation, i.e., roots of the plasma dielectric $\hat{\varepsilon}(\mathbf{k}, \omega) = 0$, for the jth unstable mode. The initial seed fluctuations are the pervasive thermal noise that is self-consistently contained in the kinetic theory [15].

The lowest order velocity-space moments of the kinetic equation gives the continuity equation

$$\frac{\partial n_s}{\partial t} + \nabla\cdot(n_s \mathbf{V}_s) = 0, \quad (4)$$

and the momentum equation

$$m_s n_s \left(\frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s\right) = n_s q_s \mathbf{E} - \nabla\cdot\mathcal{P}_s + \mathbf{R}_s, \quad (5)$$

for each species. Here, we have defined density $n_s \equiv \int d^3v f_s$, fluid flow velocity $\mathbf{V}_s \equiv \int d^3v \mathbf{v} f_s/n_s$, pressure tensor $\mathcal{P}_s \equiv \int d^3v m_s \mathbf{v}_r \mathbf{v}_r f_s$, in which $\mathbf{v}_r \equiv \mathbf{v} - \mathbf{V}_s$ is a flow-shifted phase-space variable, and the frictional force density $\mathbf{R}_s \equiv \sum_{s'} \int d^3v m_s \mathbf{v} C(f_s, f_{s'}) = \sum_{s'} \mathbf{R}^{s/s'}$. Since the collision operator conserves momentum [15], the frictional force is equal and opposite between any two species: $\mathbf{R}^{s/s'} + \mathbf{R}^{s'/s} = 0$.

To describe the ion flow in the presheath, we take the steady-state parts of Eqs. (4) and (5) and assume that the flow of each species is along the presheath electric field in the $\hat{z}$ direction. Using these one-dimensional fluid equations along with the charge density $\rho = \sum_i q_i n_i - e n_e$ in the sheath condition $d\rho/d\phi|_{\phi=0} \leq 0$ yields [8]

$$\sum_i^N \frac{n_{io}}{n_{eo}} \frac{c_{s,i}^2}{V_i^2 - v_{Ti}^2/2} \leq 1 \quad (6)$$

in which $v_{Ti}^2 = 2T_i/m_i$. Equation (6) is a generalization of the Bohm criterion to multiple ion species that was first derived by Riemann [8]. Equality typically holds [8]. In the plasmas of interest $V_i \sim c_s \gg v_{Ti}$ at the sheath edge, so the ion thermal term in Eq. (6) is typically neglected.

Previous theoretical works [9] that neglected friction between ion species, but included ion-neutral drag and ionization sources, have shown that each species should enter the sheath close to its individual sound speed $V_i \approx c_{si}$. Small deviations may occur if the ion-neutral collision frequencies are very different for different species. However, ion-neutral collisions cannot explain the experimental result that each species enters the sheath with a near common speed c_s rather than the individual sound speeds c_{si} [10–14].

Solving for the collisional friction between two Maxwellian ion species $s = 1$ and $s' = 2$ using only the Coulomb collision kernel $\mathcal{Q}_{\rm LB}$ (for stable plasmas), assuming equal temperatures ($T_1 = T_2$), and an adiabatic dielectric one finds

$$\mathbf{R}_{\rm LB}^{1-2} = \frac{\sqrt{\pi}}{2} n_1 m_1 \nu_s \frac{\bar{v}_T^3 \Delta\mathbf{V}}{\Delta V^4} \psi\left(\frac{\Delta V^2}{\bar{v}_T^2}\right) \quad (7)$$

in which ψ is the Maxwell integral,

$$\nu_s \equiv \frac{8\sqrt{\pi} q_1^2 q_2^2 n_2}{m_1^2 v_{T1}^2 \bar{v}_T} \ln\Lambda \quad (8)$$

is a reference collision frequency, $\bar{v}_T^2 \equiv v_{T1}^2 + v_{T2}^2$ and $\Delta\mathbf{V} = \mathbf{V}_1 - \mathbf{V}_2$. Aside from assuming drifting Maxwellians, the only approximation used in deriving Eq. (7) is the truncation of the k-space integral in Eq. (2) at $1/b_{\rm min}$ for large k, in the conventional fashion, so as to avoid divergence of this integral and give the $\ln\Lambda$ term. Equation (7) is plotted in Fig. 1.

Using the parameters of the example plasma, the VdV/dz term in the 1D version of Eq. (5) is $\sim c_s^2/\lambda^{i/n} \sim 10^7$ m/s^2, while $R_{\rm LB}^{1-2}/m_s n_s \sim 10^6$ m/s^2. Thus the neglect of collisional friction due to Coulomb interactions in a stable plasma was likely justified in previous theoretical

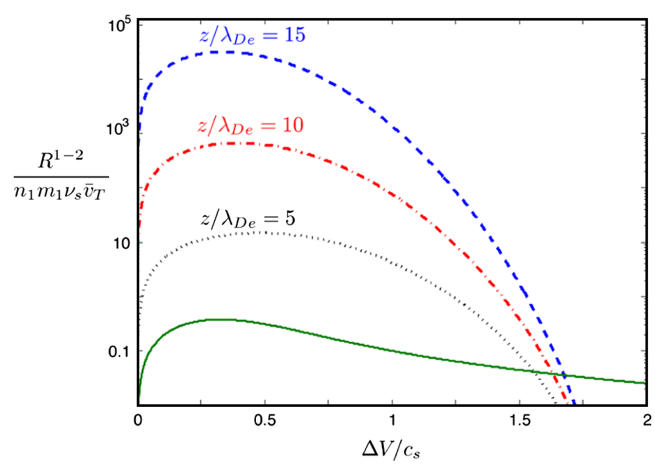

FIG. 1 (color online). Normalized collisional friction force for the parameters of [12] due to Coulomb interactions (solid line) and due to instability-enhanced collective interactions for wave growth over a distance of $z/\lambda_{\rm De} = 5$, 10 and 15 (dotted, dash-dotted and dashed lines, respectively).

works [9]. However, two-stream instabilities can lead to a significant amplification of the friction between ion species.

Calculating the instability-enhanced collisional friction requires a dispersion relation. Assuming Maxwellian distribution functions, this is obtained from the roots of $\hat{\varepsilon} = 0$ with

$$\hat{\varepsilon}(\mathbf{k}, \omega) = 1 - \sum_s \frac{\omega_{ps}^2}{k^2 v_{Ts}^2} Z'\left(\frac{\omega - \mathbf{k}\cdot\mathbf{V}_s}{k v_{Ts}}\right) \qquad (9)$$

in which Z is the plasma dispersion function. For ion waves with $k v_{Ti} \ll \omega - \mathbf{k}\cdot\mathbf{V}_s \ll k v_{Te}$, the dielectric function becomes

$$\hat{\varepsilon} = 1 + \frac{1}{k^2\lambda_{\rm De}^2} - \frac{\omega_{p1}^2}{(\omega - \mathbf{k}\cdot\mathbf{V}_1)^2} - \frac{\omega_{p2}^2}{(\omega - \mathbf{k}\cdot\mathbf{V}_2)^2}. \quad (10)$$

The dispersion relations are thus given by $\omega_j = \mathbf{k}\cdot(\mathbf{V}_1 + \mathbf{V}_2)/2 + \mathbf{k}\cdot\Delta\mathbf{V}\Omega_j$, in which the Ω_j are the four solutions of $\Omega^4 - \Omega^2(1/2 + a) - \Omega ab + 1/16 - a/4 = 0$, $a = k^2 c_s^2/[(\mathbf{k}\cdot\Delta\mathbf{V})^2(1 + k^2\lambda_{\rm De}^2)]$, and $b = (\omega_{p1}^2 - \omega_{p2}^2)/(\omega_{p1}^2 + \omega_{p2}^2)$. Two of the solutions are stable ion sound waves (with $\omega \approx kc_s$), the other two are either damped or growing ion waves [with $\omega \approx k(V_1 + V_2)$] one of which can be unstable. This unstable solution is the two-stream instability that enhances the collisional interaction. We will find that $\Omega \sim b$ and $b < 1$ (for the sample plasma parameters $b \approx 1/2$) so the Ω^4 term can be neglected for the potentially unstable root of interest. The resulting quadratic equation yields

$$\Omega = -\frac{ab \pm \sqrt{a^2b^2 + (1/2 + a)(1/4 - a)}}{1 + 2a}. \qquad (11)$$

Figure 2 shows that Eq. (11) provides an accurate approximation of the unstable root from $\hat{\varepsilon} = 0$ using Eq. (10), but we seek a further simplified form that can be used to analytically approximate $\mathbf{R}_{\rm IE}$. Noticing that $a > 1$ when $k\lambda_{\rm De} < \sqrt{c_s^2/\Delta V^2 - 1}$, we can treat a as a large number for this part of k space. Since $\Delta V \le c_{s1} - c_{s2}$ in the presheath (even in the absence of friction), this is valid for at least $k\lambda_{\rm De} \lesssim 1$ using the sample plasma parameters. In this limit, the leading term of Eq. (11) is $\Omega \approx -b/2 \pm i\sqrt{\alpha}/(1 + \alpha)$ which is unstable for all k in the range of validity. Here $\alpha = n_1 M_2/(n_2 M_1)$. When a becomes smaller than some critical value $a \le a_c$, stabilization occurs and we account for this stabilization by using the approximation $\Omega \approx -b/2 \pm i\sqrt{\alpha(1 - a_c/a)}/(1 + \alpha)$, in which a_c is obtained from Eq. (11). This gives $1/a_c = 1 + \sqrt{9 - 8b^2}$. With these, we arrive at an approximate dispersion relation for the unstable root: $\omega = \omega_R + i\gamma$, in which

$$\omega_R \approx \mathbf{k}\cdot\left(\frac{n_2}{n_e}\frac{c_{s2}^2}{c_s^2}\mathbf{V}_1 + \frac{n_1}{n_e}\frac{c_{s1}^2}{c_s^2}\mathbf{V}_2\right) \qquad (12)$$

is the real part, and

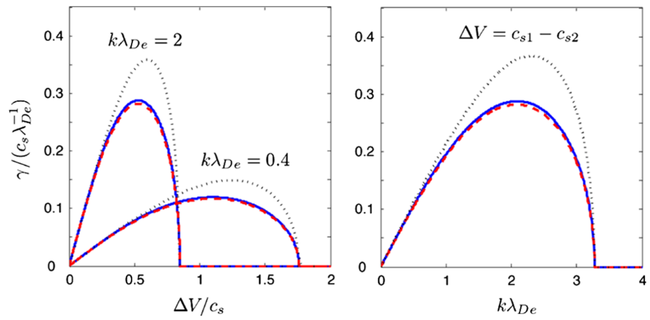

FIG. 2 (color online). Normalized growth rates calculated for the parameters of [12] from a numerical solution of Eq. (10) (solid line), from the quadratic approximation of Eq. (11) (dashed line), and from the approximation of Eq. (13) (dotted line).

$$\gamma \approx \frac{k_\parallel \Delta V\sqrt{\alpha}}{1 + \alpha}\sqrt{1 - \frac{k_\parallel^2\Delta V^2}{k^2\Delta V_{\rm up}^2}(1 + k^2\lambda_{\rm De}^2)} \qquad (13)$$

is an expression for the growth rate. The $\parallel$ direction is along $\Delta\mathbf{V}$ and $\Delta V_{\rm up}^2 = c_s^2[1 + \sqrt{1 + 32\alpha/(1 + \alpha)^2}]$ is an upper limit above which the mode stabilizes. Figure 2 shows that Eq. (13) overestimates the growth rate by as much as 30%. However, we will subsequently show that this quantitative difference does not significantly affect our central conclusion.

Next, we use Eqs. (10), (12), and (13) in Eq. (3) to compute the contribution to the collisional friction from instability-enhanced collective interactions $\mathbf{R}_{\rm IE}^{1-2}$. Since the two-stream modes are convectively unstable, the $2\gamma t$ term must be evaluated in the rest frame of the wave, as discussed in [15]. Thus, $2\gamma t = 2\int_{\mathbf{x}_0(\mathbf{k})}^{\mathbf{x}} d\mathbf{x}'\cdot\mathbf{v}_g\gamma/|\mathbf{v}_g|^2$ in which $\mathbf{v}_g \equiv \partial\omega_R/\partial\mathbf{k}$ is the group velocity, $\mathbf{x}_0(\mathbf{k})$ is the location in space where the mode $\mathbf{k}$ becomes unstable, and the integral $d\mathbf{x}'$ is taken along the path of the mode. Here we have assumed that changes due to spatial variations are weak, and we account for $\mathbf{x}_0(\mathbf{k})$ by only integrating over the unstable $\mathbf{k}$ for each spatial location $\mathbf{x}$. Following these approximations we obtain $2\gamma t \approx 2\gamma z/v_g$.

Assuming the temperatures of the ion species are equal and much colder than the electrons ($T_1 \simeq T_2 \ll T_e$), the instability-enhanced collisional friction is found to be

$$\mathbf{R}_{\rm IE}^{1-2} \simeq n_1 m_1 \nu_{12}\exp\left(W\frac{A}{2}\right)\Delta\mathbf{V} \qquad (14)$$

in which

$$\nu_{12} = \nu_s \frac{3}{160\sqrt{\pi}}\frac{\bar{v}_T\Delta V^4}{\Delta V_{\rm up}c_s^4}\frac{A^{7/2}}{4 + A^{3/2}}\frac{\alpha^{5/2}(1 + \alpha^{1/3})^2}{\alpha^2 - 1}, \qquad (15)$$

$A = \Delta V_{\rm up}^2/\Delta V^2 - 1$, and

$$W = \frac{2\sqrt{\alpha}}{(1 + \alpha)}\frac{\Delta V^2}{v_g\Delta V_{\rm up}}\frac{z}{\lambda_{\rm De}}. \qquad (16)$$

Here $v_g = (n_2 c_{s2}^2 V_1 + n_1 c_{s1}^2 V_2)/(n_e c_s^2)$ is the group velocity in the $\hat{z}$ direction of the unstable mode.

Figure 1 shows a plot of Eq. (14) for the example plasma of Ref. [12] after the two-stream instabilities have grown for $z/\lambda_{De} = 5$, 10 and 15. We have used for the group speed $v_g = c_s$. The presheath length scale for this plasma is $l \sim 5$ cm and $\lambda_{De} \sim 6 \times 10^{-3}$ cm [12], so the wave growth distances shown in Fig. 1 are much shorter than the presheath length $z/l \sim 10^{-2}$. Since a tenfold enhancement of $\mathbf{R}$ over the stable plasma level is required for friction to become important, and for $z/\lambda_{De} = 15$, the enhancement is over 10^4, the distance unstable waves must grow before the instability-enhanced friction dominates the momentum balance equation is much shorter than the presheath length scale [even accounting for the $\lesssim 30\%$ error introduced by the approximation of γ from Eq. (13)]. This shows that in the cold ion limit, the collisional friction between ion species is so strong that each species should continually have approximately the same speed throughout the presheath, and in particular at the sheath edge. Thus, the only solution to Eq. (6) is that each species obtain the system sound speed c_s at the sheath edge, as has been suggested in the experimental literature [10–14].

When finite ion temperature effects are included, instability onset occurs when $\Delta V \geq \Delta V_c \sim \mathcal{O}(v_{Ti})$. This produces a small correction (for the plasmas of interest) to the result that each ion species obtains the common c_s at the sheath edge. To estimate ΔV_c, one can repeat the procedure above using the fluid plasma dispersion relation, which is Eq. (10) with the denominator of the ion thermal terms replaced by $(\omega - \mathbf{k} \cdot \mathbf{V}_s)^2 - v_{Ts}^2/2$ [14]. The lowest order equation gives $\gamma = \sqrt{\alpha[(\mathbf{k} \cdot \Delta \mathbf{V})^2 - k^2 \Delta V_c^2]}/(1 + \alpha)$ which is the growth rate if $\Delta V > \Delta V_c k/k_\parallel$ where

$$\Delta V_c \equiv \sqrt{\frac{1 + \alpha}{2\alpha}} \sqrt{v_{T1}^2 + \alpha v_{T2}^2} \tag{17}$$

is a critical minimum relative flow speed.

Thus the relative flow speed will effectively have a maximum value of $\Delta V = \Delta V_c$, rather than zero. Using this condition, along with the generalized form of Bohm's criterion from Eq. (6), uniquely determines the speed of each ion species at the sheath edge. For the limit $v_{T,i} \ll c_{s,i}$ this reduces to

$$V_1 \simeq c_s + \frac{n_2}{n_e} \frac{c_{s2}^2}{c_s^2} \Delta V_c \quad \text{and} \quad V_2 \simeq c_s - \frac{n_1}{n_e} \frac{c_{s1}^2}{c_s^2} \Delta V_c. \tag{18}$$

Thus, in typical gas discharge plasmas, ions fall into the sheath near the sound speed c_s rather than near their individual sound speeds $c_{s,i}$.

For the parameters of [12], we calculate from $\Delta V = \Delta V_c$ and Eq. (6), assuming equality, that $V_{Ar} = 1180$ m/s and $V_{Xe} = 830$ m/s. The individual sound speeds are $c_{s,Ar} = 1290$ m/s and $c_{s,Xe} = 710$ m/s, and the system sound speed is $c_s = 1040$ m/s. The experiment measured the rms speed of each species with laser-induced fluorescence to be $v_{rms,Ar} = 1100 \pm 60$ m/s and $v_{rms,Xe} = 940 \pm 50$ m/s. It should be noted that the rms speed is not the fluid moment speed that we calculate. The difference is $\mathcal{O}(v_{Ti})$, so this comparison does not accurately resolve finite ion temperature effects. Nonetheless, accounting for instability-enhanced collisional friction gives results consistent with the measurements to this accuracy, whereas other theories do not.

This material is based upon work supported by the National Science Foundation Graduate Research Program (S. D. B.) and by U.S. Department of Energy Grant No. DE-FG02-86ER53218. We thank Dr. Noah Hershkowitz and Dr. Greg Severn for helpful discussions.

*sdbaalrud@wisc.edu

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